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Subject Code: **ME-402**

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UNIT-2

Dynamic characteristics:

a) Speed of response and measuring lag, b) Fidelity and dynamic error, c) Over shoot, d) Dead time and dead zone, e) Frequency response.

a) Speed of response and measuring lag: In a measuring instrument the speed of response (or) responsiveness is defined as the rapidity with which an instrument responds to a change in the value of the quantity being measured.

Measuring lag refers to delay in the responds of an instrument to a change in the input signal. The lag is caused by conditions such as inertia, or resistance.

b) Fidelity and dynamic errors: Fidelity of an instrumentation system is defined as the degree of closeness with which the system indicates (or) records the signal which is upon its. It refers to the ability of the system to reproduce the output in the same form as the input. If the input is a sine wave then for 100% fidelity the output should also be a sine wave. The difference between the indicated quantity and the true value of the time quantity is the dynamic error. Here the static error of instrument is assumed to be zero.

c) Over shoot: Because of maximum and inertia. A moving part i.e., the pointer of the instrument does not immediately come to rest in the final deflected position. The pointer goes beyond the steady state i.e., it over shoots.

The over shoot is defined as the maximum amount by which the pointer moves beyond the steady state.

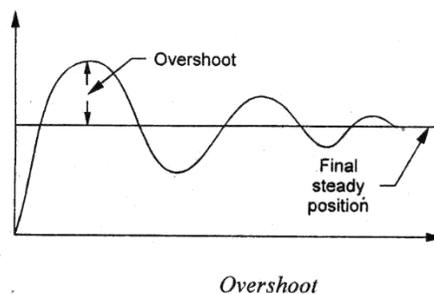


Figure.1 Overshoot

d) dead time and dead zone: Dead time is defined as the time required for an instrument to begin to respond to a change in the measured quantity it represent the time before the instrument begins to respond after the measured quantity has been altered.

Dead zone define the largest change of the measured to which the instrument does not respond. Dead zone is the result as friction backlash in the instrument.

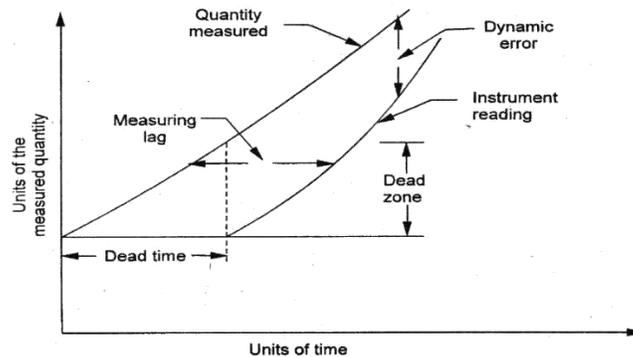


Figure.2 Dynamic terms

e) Frequency response: (The dynamic performance of both measuring and control system is determine by applying some known and predetermined input signal to its primary sensing element and them) Maximum frequency of the measured variable that an instrument is capable of following with error. The usual requirement is that the frequencies of the measured should not exceed 60% of the natural frequency measuring instrument.

Standard test inputs: The dynamic performance of both measuring and control system is determined by applying some known and predetermined input signal to its primary sensing element and then studying the behavior of the output signals.

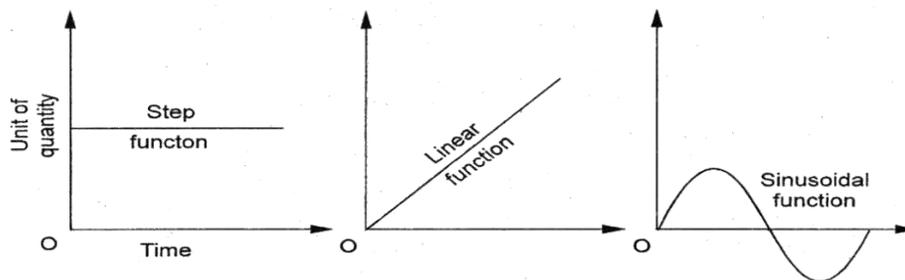


Figure.3 Standard input function

The most common standard inputs used for dynamic analysis

1. Step functions
2. Linear (or) ramp functions
3. Sinusoidal (or) sine wave functions

Static and dynamic response: The static characteristics of measuring instruments are concerned only with the steady-state reading that the instrument settles down to, such as accuracy of the reading. The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets only apply when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected. In any linear, time-invariant measuring system, the following general relation can be written between input and output for time $(t) > 0$:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i \quad (1)$$

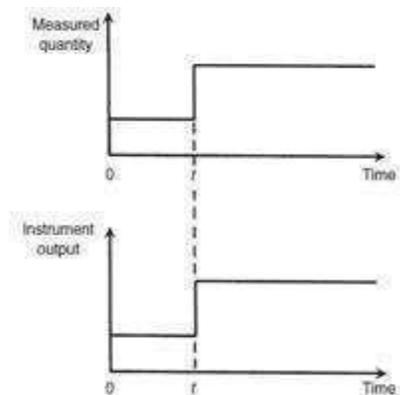
Where q_i is the measured quantity, q_o is the output reading, and $a_0 \dots a_n, b_0 \dots b_m$ are constants. If we limit consideration to that of step changes in the measured quantity only, then Equation (2) reduces to

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2)$$

Zero-Order Instrument

$$a_0 q_o = b_0 q_i \quad \text{or} \quad q_o = b_0 q_i / a_0 = K q_i \quad (3)$$

If all the coefficients $a_1 \dots a_n$ other than a_0 in Equation (2) are assumed zero, then where K is a constant known as the instrument sensitivity as defined earlier. Any instrument that behaves according to Equation (3) is said to be of a zero-order type. Following a step change in the measured quantity at time t , the instrument output moves immediately to a new value at the same time instant t , as shown in Figure. A potentiometer, which measures motion is a good example of such an instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.



First-Order Instrument: If all the coefficients $a_2 \dots$ an except for a_0 and a_1 are assumed zero in Equation

$$(2) \text{ then } a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (3)$$

Any instrument that behaves according to Equation (4) is known as a first-order instrument. If d/dt is replaced by the D operator in Equation (4), we get

$$a_1 D q_o + a_0 q_o = b_0 q_i \quad (4)$$

$$q_o = \frac{(b_0/a_0) q_i}{[1 + (a_1/a_0) D]} \quad (5)$$

Defining $K \approx b_0/a_0$ as the static sensitivity and $\tau \approx a_1/a_0$ as the time constant of the system,

Equation (5) becomes  $q_o = \frac{K q_i}{1 + \tau D} \quad (6)$

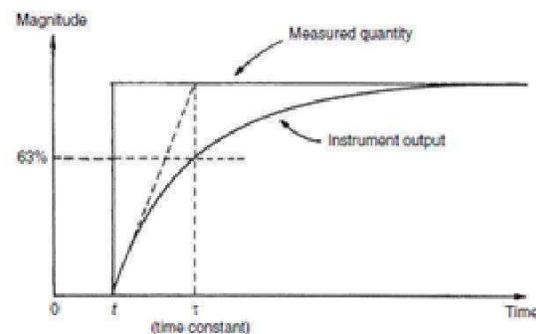
Second-Order Instrument: If all coefficients $a_3 \dots$ other than a_0 , a_1 , and a_2 in Equation (2) are assumed zero, then we get

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

$$a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i$$

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2}$$

$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad (7) \quad \zeta = a_1/2\sqrt{a_0 a_2}$$



$$\zeta = \frac{a_1}{2a_0 \sqrt{a_2/a_0}} = \frac{a_1 \omega}{2a_0} \quad (8)$$

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \quad (9)$$

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right)D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

$$\frac{q_o}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1}$$

This is the standard equation for a second-order system, and any instrument whose response can be described by it is known as a second-order instrument. If Equation (9) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter ξ . The output responses of a second-order instrument for various values of ξ following a step change in the value of the measured quantity at time t are shown in Figure.

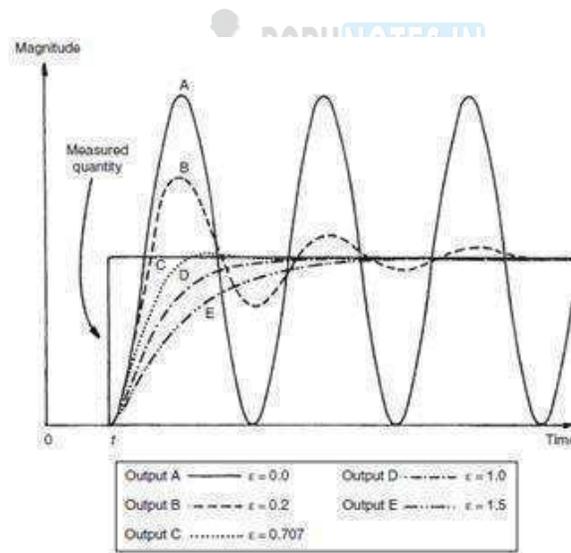


Figure.4 Second Order Response

Commercial second-order instruments, of which the accelerometer is a common example, are generally designed to have a damping ratio (ξ) somewhere in the range of 0.6–0.8.

The Mean and Mode

The *sample mean* is the average and is computed as the sum of all the observed outcomes from the sample divided by the total number of events. We use \bar{x} as the symbol for the sample mean. In math terms,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x$$

.....where n are the sample size and the x corresponding to the observed valued.

Example

Suppose you randomly sampled six acres in the Desolation Wilderness for a non-indigenous weed and came up with the following counts of this weed in this region:

34, 43, 81, 106, 106 and 115

We compute the sample mean by adding and dividing by the number of samples, 6.

$$\frac{34 + 43 + 81 + 106 + 106 + 115}{6} = 80.83$$

We can say that the sample mean of non-indigenous weed is 80.83.

The *mode* of a set of data is the number with the highest frequency. In the above example 106 is the mode, since it occurs twice and the rest of the outcomes occur only once.

The *population mean* is the average of the entire population and is usually impossible to compute. We use the Greek letter μ for the population mean.

Variance, Standard Deviation and Coefficient of Variation

The mean, mode, median, and trimmed mean do a nice job in telling where the center of the data set is, but often we are interested in more. For example, a pharmaceutical engineer develops a new drug that regulates iron in the blood. Suppose she finds out that the average sugar content after taking the medication is the optimal level. This does not mean that the drug is effective. There is a possibility that half of the patients have dangerously low sugar content while the other half has dangerously high content. Instead of the drug being an effective regulator, it is a deadly poison. What the pharmacist needs is a measure of how far the data is spread apart. This is what the variance and standard deviation do. First we show the formulas for these measurements. Then we will go through the steps on how to use the formulas.

We define the *variance* to be

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x - \bar{x})^2$$

.....and the *standard deviation* to be

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x - \bar{x})^2}$$

Variance and Standard Deviation: Step by Step

1. Calculate the mean, \bar{x} .
2. Write a table that subtracts the mean from each observed value.
3. Square each of the differences.
4. Add this column.
5. Divide by $n-1$ where n is the number of items in the sample this is the variance.
6. To get the *standard deviation* we take the square root of the variance.



Example

The owner of the Chas Tahoe restaurant is interested in how much people spend at the restaurant. He examines 10 randomly selected receipts for parties of four and writes down the following data.

44, 50, 38, 96, 42, 47, 40, 39, 46, 50

He calculated the mean by adding and dividing by 10 to get

$$\bar{x} = 49.2$$

Below is the table for getting the standard deviation:

x	$x - 49.2$	$(x - 49.2)^2$
44	-5.2	27.04
50	0.8	0.64
38	11.2	125.44
96	46.8	2190.24

42	-7.2	51.84
47	-2.2	4.84
40	-9.2	84.64
39	-10.2	104.04
46	-3.2	10.24
50	0.8	0.64
Total		2600.4

Now

$$\frac{2600.4}{10 - 1} = 288.7$$

Hence the variance is 289 and the standard deviation is the square root of 289 = 17.

Since the standard deviation can be thought of measuring how far the data values lie from the mean, we take the mean and move one standard deviation in either direction. The mean for this example was about 49.2 and the standard deviation was 17. We have:

$$49.2 - 17 = 32.2;$$

$$49.2 + 17 = 66.2$$

What this means is that most of the patrons probably spend between \$32.20 and \$66.20.

The sample standard deviation will be denoted by s and the population standard deviation will be denoted by the Greek letter σ .

The sample variance will be denoted by s^2 and the population variance will be denoted by σ^2 .

The variance and standard deviation describe how spread out the data is. If the data all lies close to the mean, then the standard deviation will be small, while if the data is spread out over a large range of values, s will be large. Having outliers will increase the standard deviation.

One of the flaws involved with the standard deviation, is that it depends on the units that are used. One way of handling this difficulty, is called the *coefficient of variation* which is the standard deviation divided by the mean times 100%

$$CV = \frac{s}{x} 100\%$$

In the above example, it is

$$\frac{17}{49.2} \times 100\% = 34.6\%$$

This tells us that the standard deviation of the restaurant bills is 34.6% of the mean.

The following steps show how to calculate average deviation for the mean. If you want to calculate average deviation for the median, just replace any value for the mean with the value for the median.

The absolute deviation formula (i.e. the formula to calculate the distance for one point) is:

$$\text{Absolute deviation} = |x - \bar{x}|$$

Which leads to the average deviation formula:

$$Dx = (|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|) / N$$

Sample question: Find the average deviation of the following set of numbers: 3, 8, 8, 8, 8, 9, 9, 9, and 9.

Step 1: Find the mean:

$$(3 + 8 + 8 + 8 + 8 + 9 + 9 + 9 + 9) = 71.9 = 7.89.$$

Step 2: Find each individual absolute deviation using the formula $|x - \bar{x}|$.

$$|3 - 7.89| = 4.89$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|9 - 7.89| = 1.11$$

$$|9 - 7.89| = 1.11$$

- $|9 - 7.89| = 1.11$

- $|9 - 7.89| = 1.11$

Step 3: Add up all of the values you found in Step 1.

$$4.89 + 0.11 + 0.11 + 0.11 + 0.11 + 1.11 + 1.11 + 1.11 + 1.11 = 9.77$$

Step 4: Divide by the number of items in your data set. There are 9 items, so:

$$9.77 / 9 = 1.09.$$

The average deviation is 1.09.

Normal Distribution

A normal distribution, sometimes called the bell curve, is a distribution that occurs naturally in many situations. For example, the bell curve is seen in tests like the SAT and GRE. The bulk of students will score the average (C), while smaller numbers of students will score a B or D. An

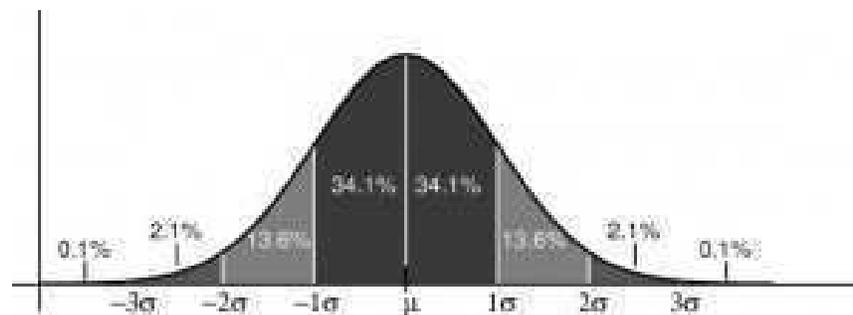
even smaller percentage of students score an F or an A. This creates a distribution that resembles a bell (hence the nickname). The bell curve is symmetrical. Half of the data will fall to the left of the mean; half will fall to the right.

Many groups follow this type of pattern. That's why it's widely used in business, statistics and in government bodies like the FDA:

- Heights of people.
- Measurement errors.
- Blood pressure.
- Points on a test.
- IQ scores.
- Salaries.

The empirical rule tells you what percentage of your data falls within a certain number of standard deviations from the mean:

- 68% of the data falls within one standard deviation of the mean.
- 95% of the data falls within two standard deviations of the mean.
- 99.7% of the data falls within three standard deviations of the mean.



The standard deviation controls the spread of the distribution. A smaller standard deviation means that the data is tightly clustered around the mean; the normal distribution will be taller. A larger standard deviation means that the data is spread out around the mean; the normal distribution will be flatter and wider.

Properties of a normal distribution

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

Method of Least Squares

Suppose that we are given a data set of n observations from an experiment. Say that we are interested in fitting a straight line

$$y = ax + b$$

to the given data. Find the ' n ' residuals e_i by:

$$e_i = y_i - (ax_i + b), \quad i = 1, 2, \dots, n \quad (2)$$

Now consider the sum of the squares of e_i i.e

$$\begin{aligned}
 E &= \sum_{i=1}^n e_i^2 \\
 &= \sum_{i=1}^n [y_i - (ax_i + b)]^2 \quad (3)
 \end{aligned}$$

Note that E is a function of parameters a and b . We need to find a, b such that E is minimum. The necessary condition for E to be minimum is given by:

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = 0 \quad (4)$$

The condition $\frac{\partial E}{\partial a} = 0$ yields:

$$\frac{\partial E}{\partial a} = \sum_{i=1}^n 2x_i[y_i - (ax_i + b)] = 0$$

$$a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i = \sum_{i=1}^n x_i y_i \quad (5)$$

i.e

Similarly the condition $\frac{\partial E}{\partial b} = 0$ yields

$$a \sum_{i=1}^n x_i + nb = \sum_{i=1}^n y_i \quad (6)$$

Equations (5) and (6) are called as normal equations, which are to be solved to get desired values for a and b.

The expression for E i.e (3) can be re-written in a convenient way as follows:

$$E = \left(\sum_{i=1}^n y_i^2 - a \sum_{i=1}^n x_i y_i - b \sum_{i=1}^n y_i \right) \quad (7)$$

Example: Using the method of least squares, find an equation of the form

$y = ax + b$ That fits the following data:

x	0	1	2	3	4
y	1	5	10	22	38

Solution: Consider the normal equations of least square fit of a straight line i.e

$$a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i = \sum_{i=1}^n x_i y_i \quad (1)$$

$$a \sum_{i=1}^n x_i + nb = \sum_{i=1}^n y_i \quad (2)$$

Here $n = 5$.

From the given data, we have,

x	Y	xy	x ²
0	1	0	0
1	5	5	1
2	10	20	4
3	22	66	9
4	38	152	16

$$\sum_i x_i = 10 \quad \sum_i y_i = 76 \quad \sum_i x_i y_i = 243 \quad \sum_i x_i^2 = 30$$

Therefore the normal equations are given by:

$$30a + 10b = 243 \dots\dots\dots (3)$$

$$10a + 5b = 76 \dots\dots\dots (4)$$

On solving (3) and (4) we get

$$a = 9.1, b = -3 \dots\dots\dots (5)$$

Hence the required fit for the given data is

$$y = 9.1x - 3 \dots\dots\dots (6)$$

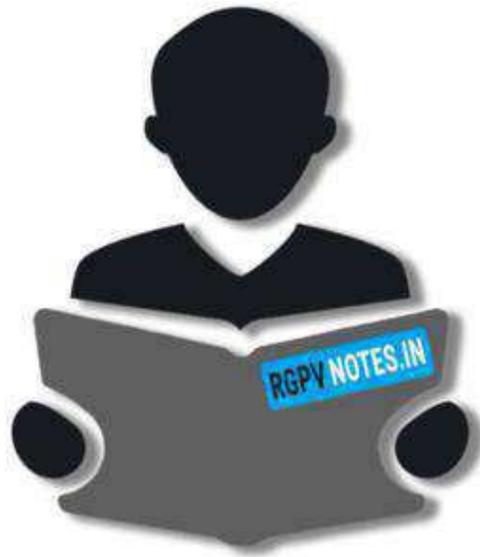
Uncertainty Analysis

Experimental uncertainty analysis is a technique that analyses a *derived* quantity, based on the uncertainties in the experimentally *measured* quantities that are used in some form of mathematical relationship ("model") to calculate that derived quantity. The model used to convert the measurements into the derived quantity is usually based on fundamental principles of a science or engineering discipline.

The uncertainty has two components, namely, bias (related to *accuracy*) and the unavoidable random variation that occurs when making repeated measurements (related to *precision*). The measured quantities may have biases, and they certainly have random variation, so what needs to be addressed is how these are "propagated" into the uncertainty of the derived quantity. Uncertainty analysis is often called the "propagation of error."

It will be seen that this is a difficult and in fact sometimes intractable problem when handled in detail. Fortunately, approximate solutions are available that provide very useful results, and these approximations will be discussed in the context of a practical experimental example.





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